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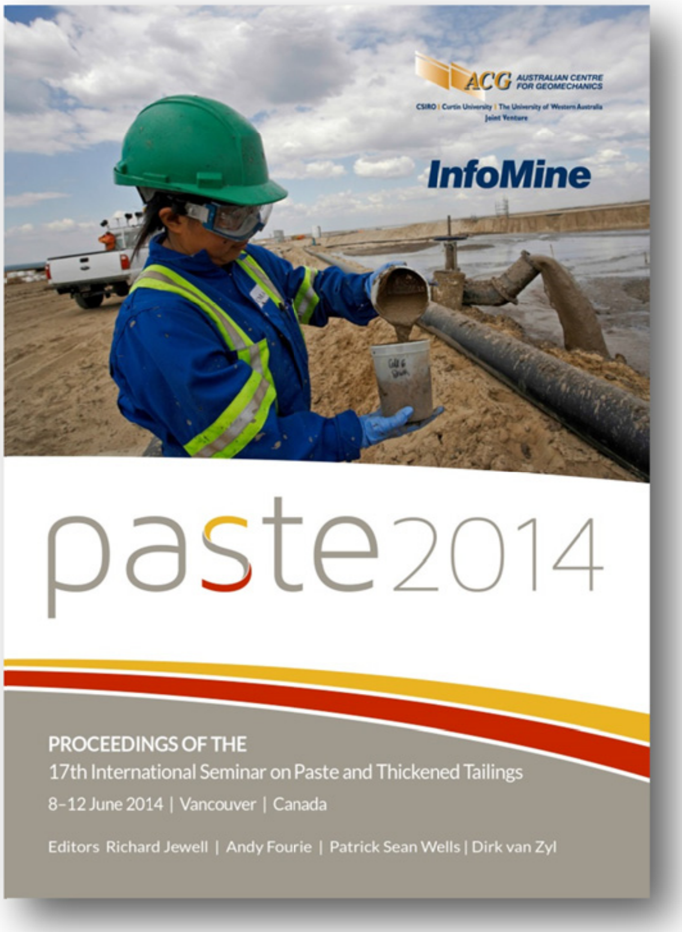

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Modified beach slope prediction model for non-segregating thickened tailings

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Abstract

This paper is an extension to a previous theoretical model developed for non-segregating thickened tailings beach slope prediction. The original model was presented by two of the authors at Paste 2007 (Pirouz and Williams, 2007). The visual observations and experimental data from tilting flume testing at three different mine sites have been analysed and the findings are used to modify the theoretical model to be applicable to a wide range of slurries with different rheological and flow conditions. The overall structure of the new model is similar to the originally developed model and is based on the theory that, on a large scale, the slope of the overall tailings beach is determined by the slope of self-formed tailings channels.

A modified Hedstrom Number (He) for tailings slurries flow has been introduced in this paper for the first time. The experimental data from flow-through tilting flume tests show that a strong correlation between the Reynolds Number (Re) and the Hedstrom Number exists for the flow condition at the equilibrium slope.

This unique relation between the two dimensionless numbers defines the condition of flow in the self- formed channel and can be used to define the minimum required velocity (or minimum Reynolds number) required to achieve the total transport condition in channel flow.

A modified model for the shape of the self-formed channel based on the stability of individual particles on the bank of the channel is also presented.

The validity of the available friction factor for calculation of head loss in non-Newtonian fluid has been investigated for the open channel flow of thickened tailings and a friction factor which is applicable for a wide range of Reynolds number (from 120 to 10,000) and Hedstrom number (from 630 to 45,500) is defined.

A minimum required velocity (critical velocity) for total transport in channel flow has also been defined as a function of Reynolds number at equilibrium slope.

Based on the new findings, the original beach slope prediction model has been modified and re-presented here. This modified model replaces the Re-He approach for beach slope prediction, originally presented in 2007.

1 Introduction

The thickened tailings stacking concept was first introduced to the mining industry as an alternative method to conventional tailings disposal methods in the 1970s by Eli Robinsky in Canada and then taken up by Paul Williams in Australia in the 1980s (Williams, 2009).

Considering the thickened tailings disposal method in the form of down valley discharge (DVD) or central thickened discharge (CTD) as a potential solution for the management of mine tailings has become more popular among mine owners in the last 10 years due to some of the obvious advantages of the method, such as water saving and increasing storage efficiency (reducing embankment height), which contribute directly to reducing the overall operating costs.

One of the key parameters that needs to be evaluated at a very early stage of the study for the feasibility of stacked tailings disposal schemes (DVD or CTD) at any site is the achievable beach slope.

Incorrectly estimating the achievable beach slope at the design stage of a thickened tailings scheme (especially over-estimation of the slope) is likely to have severe consequences and will impact on the economy of the project for the entire life of the mine.

A literature review on the subject (Pirouz and Williams, 2007) shows that despite considerable effort in attempting to understand the tailings beach formation process and the prediction of beach slope, due to the complex nature of the phenomena, no comprehensive theory or model exists that is accepted by all researchers, and the topic has remained as one of the subjects that still requires further research.

Among different theories that exist for thickened tailings beach formation and development, the equilibrium channel slope model, which was originally developed by Williams and Meynink (1986) and re-stated by Williams (2001) and is based on visual observations from real size tailings stacks, seems to be the most relevant one and has been adopted in the present research.

2 Principles and theory

It is a matter of common observational experience that when a continuous flow of thickened tailings is discharged over a surface (natural ground, previously dried tailings beach or experimental flume), the flow initially spreads itself out in the form of sheets and fans, which provide a bed of fresh tailings.

As more time passes and the tailings flow continues, and a bed layer of sufficient thickness is created from newly deposited tailings, a narrow self-formed channel quite suddenly appears within the new, now stationary laminar sheet, and the flow depth and velocity increase within this new channel. The self-formed channel development on a stack is shown in Figure 1.

Figure 1 Development of self-formed channels on a thickened tailings stack

The description given above is the basis for the equilibrium channel slope model. This is described in more detail in Pirouz et al. (2005), Pirouz and Williams (2007) and Pirouz et al. (2013). The model is based on the proposition that the overall beach slope of a stack formed by non-segregating tailings is determined by the limiting equilibrium slope of self-formed channels. The self-formed channels are of optimum shape to minimise the energy required for the transport (minimum bed slope) and the tailings flow in the self-formed channels is steady state and total transport flow.

Based on the above concept the problem of predicting the tailings beach slope is reduced to being able to predict the channel shape and minimum slope required for a self-formed channel to carry the tailings slurry in a steady state, total transport condition with no solids deposition or bed erosion.

2.1 Segregating and non-segregating slurries

To use the advantage of tailings beach slope to maximise the storage capacity in the thickened tailings stacking method, it is essential that the slurry is discharged to the storage facility with a consistency above the segregation threshold limit (STL) (Williams, 2001).

Experimental testwork and observation (Pirouz et al., 2008) shows that for a particular slurry there is a specific consistency (solids content) beyond which no segregation occurs during the settling process in a static settling column and all coarse and fines particles settle together in a zone. This limit is called the static segregation threshold limit (SSTL).

However, if the sample is sheared during the settling process, additional segregation can occur. If the slurry sample is thickened to a higher consistency, then a higher level of shearing will be required to cause the sample to segregate. If this process of thickening and applying shear is progressed, a certain level of consistency (solids content) can be found for any slurry at which the shearing will no longer cause segregation in the slurry during settling (Pirouz et al., 2008). This consistency is called the sheared (or dynamic) segregation threshold limit (DSTL).

Continuing to apply a higher level of shearing to a slurry which is at or above the DSTL will cause just enough eddies (or mixing) to form in the sample to prevent the settling of particles. This process is shown in Figure 2 for a slurry sample tested in the laboratory. The apparatus used a rotational mixer to apply shear.

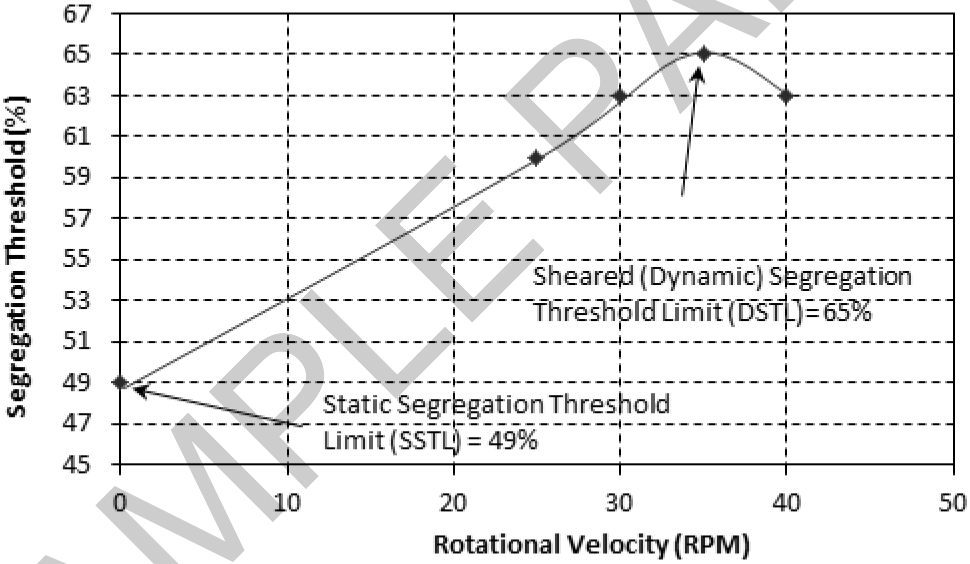


Figure 2 Static and sheared (dynamic) segregation threshold limit for slurry sample

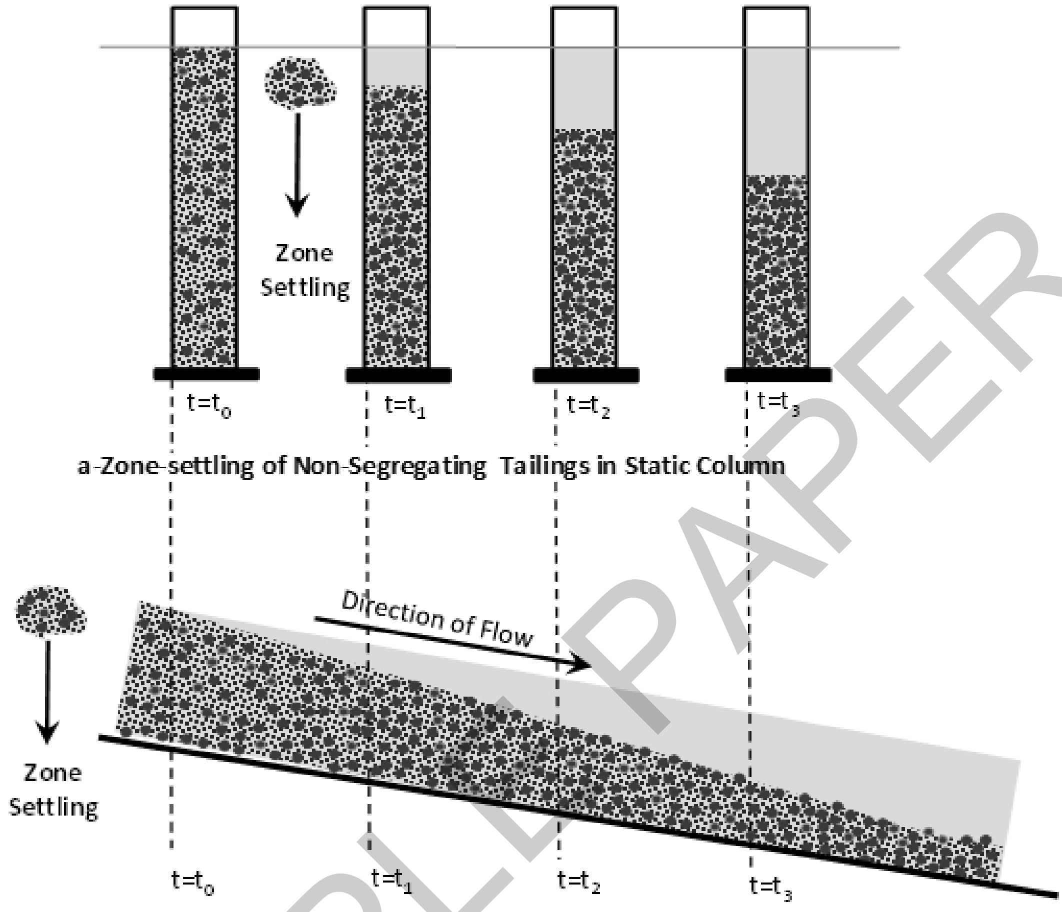
Experimental test results are also available from slurry flow in an open channel that supports the above behaviour (Pirouz et al., 2008 and 2013).

Slurries that are thickened to (or beyond) DSTL may be considered non-segregating slurries. For slurries that are thickened to (and beyond) SSTL but are still below the DSTL, depending on slurry characteristics such as particle size distribution (PSD) and rheology, theoretically, some level of segregation (possibly not more than a small percentage of the coarsest particle) can be expected if the slurry is subjected to shearing.

For thickened tailings stacking, it is required that the tailings slurry to be thickened to SSTL as a minimum, and ideally to DSTL and beyond.

2.2 Flow condition in a self-formed channel

Zone settling, in which all particles settle together in zones, is encountered in non-segregating thickened slurry mixtures and is recognised as the main mechanism for density profile formation in a static settling column and for laminar flow of non-segregating slurries. As in the laminar regime there is no upward component of velocity or momentum to counterbalance the zone settling of the particles, the formation of a density gradient with depth in the laminar regime is inevitable. In fact shearing applied to the slurry in motion facilitates the settling of particles. This process is illustrated in Figure 3.



a-Zone-settling of Non-Segregating Tailings in Static Column

b- Zone-settling of Non-Segregating Tailings in Laminar Flow in a channel

Figure 3 **Zone-settling in static settling column and laminar flow**

As soon as the slurry layer closest to the channel bed reaches a certain consistency (as a result of density profile formation with depth) the channel slope will not be sufficient to carry the layer any further and the bottom layer stops moving with the rest of the flow. The same happens progressively to the overlaying layers. Therefore laminar flow cannot persist over long distances, since zone settling within the flowing slurry layers will lead to deposition of the slurry after a finite period of time. This process will result in gradual steepening of the bed-slope until a point is reached when a certain level of turbulence appears in the flow. At this point the flow confines itself into a self-formed channel in which the required level of turbulence for overcoming the zone settling of particles exists (Pirouz et al., 2013).

It is postulated that the slope of the channel will be steep enough for the generated turbulence to just overcome the zone settling and to prevent the density gradient with depth from forming.

It is worth mentioning that as reported by Pirouz and Williams (2007), because the slurry is thickened to a high solids concentration the zone-settling velocity is low and hence the level of turbulence that is needed to counterbalance the zone-settling velocity is expected to be small as well.

If the slurry is thickened to or beyond DSTL, the slurry flow in the self-formed channel has to be total transport flow (i.e., all particles are transported by the flow).

This is also supported by experimental evidence. The test results showed that samples taken from the top and bottom of the flow at different locations along the length of the flume had the same PSD (Pirouz et al., 2008 and 2013).

2.3 Total transport mechanism in a self-formed channel

Based on observations and experimental evidence (Pirouz et al., 2005 and 2013), two different transport mechanisms can be recognised in self-formed tailings channels as the solids content and rheological resistance of a non-segregating slurry increases: turbulent flow (without plug formation), and plug flow. Figure 4 shows these two mechanisms.

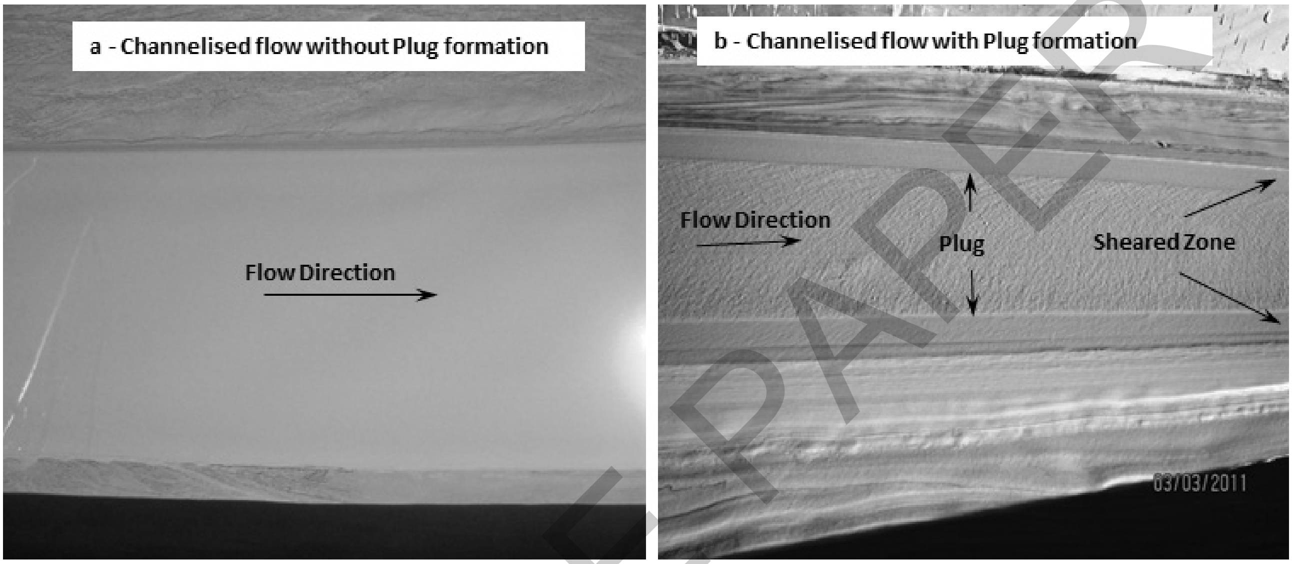


Figure 4 Different mechanisms of total transport of solids in channelised slurry flow

The formation of a central plug in the flow of a non-Newtonian fluid is a direct result of the existence of yield stress in the fluid.

All thickened tailings slurries exhibit a yield stress and their rheological characteristics can be described by the well-known Herschel-Bulkley model for yield pseudoplastic materials as below:

$$\tau = \tau_y + K\dot{\gamma}^n \tag{1}$$

Where τ is the shear stress in Pa, τ_y is the slurry yield stress in Pa, $\dot{\gamma}$ is shear rate in s^{-1}, K is the slurry consistency factor in $\mathrm{Pa.s^n}$ and n is the slurry behaviour index (smaller than unity).

The existence of the plug and sheared zone in the pipe flow of non-Newtonian fluids has previously been recognised by other researchers, and some advances in the theoretical approach to analysis of this type of non-Newtonian pipe flow has been made. Some of the earliest publications on the subject are the papers by Hedstrom (1952), Hanks (1963), Hanks and Pratt (1967) and Slatter (1997).

The plug is simply the region of the flow where the applied shear to the slurry particles is smaller than the yield stress of the slurry. Figure 5 shows the details of the plug flow model for slurry flow in an open channel.

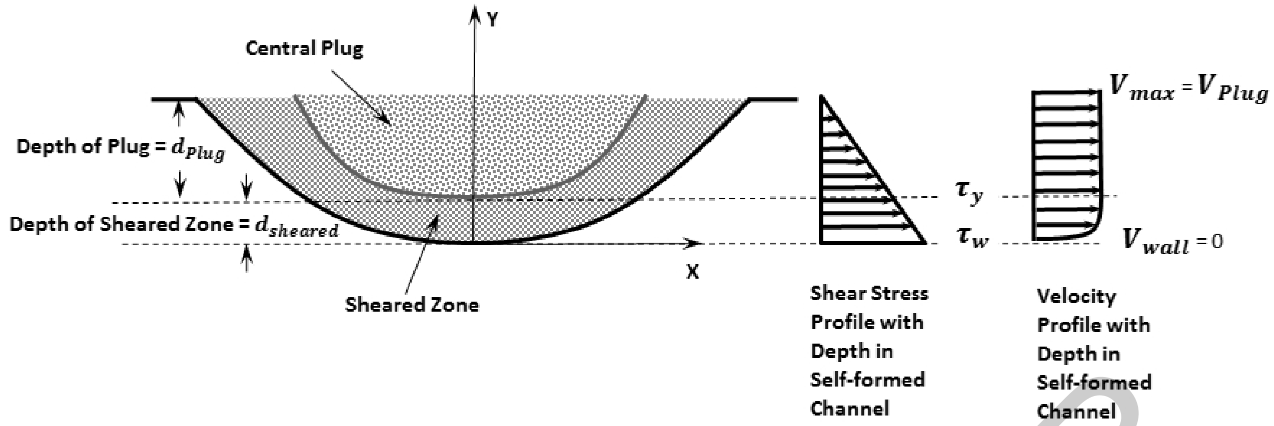


Figure 5 Plug flow model for thickened tailings slurries in self-formed channel

The following equation defines plug flow in an open channel based on the originally developed equation for plug flow in pipes (Hanks, 1963):

$$\frac{d_{plug}}{d_{total}} = \frac{\tau_y}{\tau_w} \tag{2}$$

Where τ_w is the wall shear stress, d_{plug} is the depth of the plug and d_{total} is the total depth of the flow.

The wall shear stress in open channel flow is described by DuBoys' equation as follow (Abulnaga, 2002):

$$\tau_w = \rho_m R_H S \tag{3}$$

Where ρ_m is the slurry density, R_H is the hydraulic radius of channel and S is the channel slope.

In the case of a self-formed channel, S is the equilibrium slope that creates steady state, total transport flow conditions in the channel. The bed slope, S, determines the flow velocity and hence the turbulence level in the channel.

From Equations (2) and (3) it can be seen that the formation and the size of the plug depend on the yield stress of the slurry on one hand, and on the other hand they depend on the level of turbulence (in other words, flow velocity, i.e. bed slope) that is required to counterbalance the zone-settling velocity of the particles to establish steady state total transport flow.

Depending on the rheology of the slurry (yield stress) and the zone-settling velocity of the particles, both of which determine the level of required turbulence (bed slope), the equilibrium slope for one slurry may mean a plug flow regime, while for another slurry it may mean a turbulent regime with negligible or no plug at all.

The sheared zone is subjected to the high shear rate applied due to the relatively homogeneous moving plug on one side and the stationary boundaries (channel wall) on the other. Therefore, for the reasons explained in Section 2.2, for the plug flow regime to persist as a transport mechanism in a self-formed channel over the length of the beach, the flow regime in the sheared zone cannot be purely laminar and a certain level of turbulence must exist in this zone.

The available theories and models for the analysis of the plug flow (Slatter, 1997) are all developed for pipe flow where the shear annulus is symmetrical and are based on the assumption that the flow in the sheared zone is laminar. The wall shear stress value, $\tau_{w,}$ is one of the most important inputs to the existing plug flow models. Available analytical models developed from the laminar flow theory are not able to estimate τ_w accurately for the case of channel flow in equilibrium slope.

For pipe flow, τ_w can be estimated from pipe loop testing, but for open channel flow, as seen from equation (3), the channel slope needs to be known.

For the above reasons it is concluded by the authors that until further progress is made on the subject, the plug flow models currently available cannot be applied to channel flow at equilibrium slope, and hence they have not been considered in the current study.

2.4 Self-formed channel shape

The development of self-formed channels on the stack only occurs if the tailings are discharged continuously. The channelisation starts with a relatively wide and shallow channel. As time passes, the tailings flow gradually adjusts the shape and slope of this initial channel by depositing tailings solids on the bed and banks to form a narrower and deeper channel. This process will continue until the self-formed channel finds its optimum unique shape and slope. Turbulent flow will occur in this final channel. With this optimum shape and slope, minimum energy is used for tailings flow down the beach. This process is shown in Figure 6.

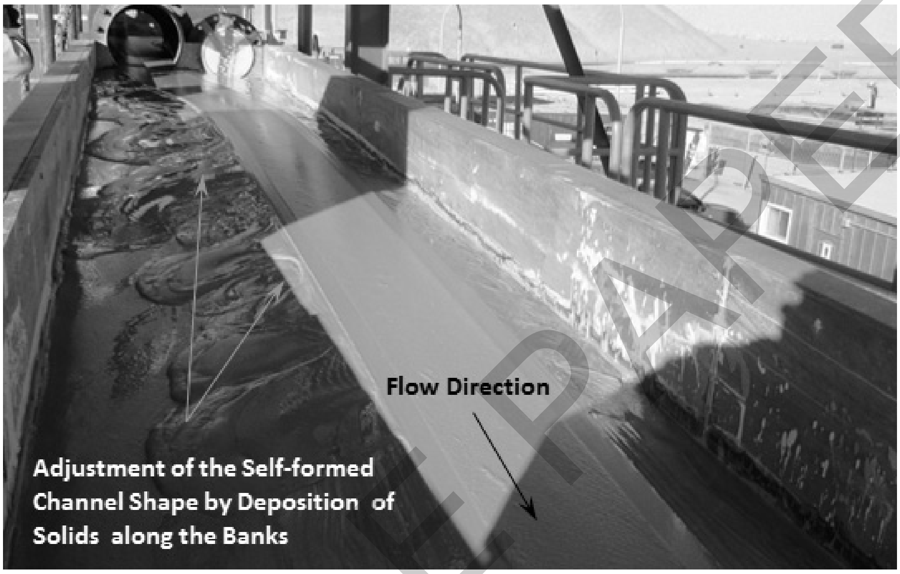


Figure 6 Self-formed channel formation and development

As is known from basic hydraulics, the best hydraulic section is semicircular in shape and has the maximum cross-sectional area for a minimum wetted perimeter (minimum resistance to flow) (Chow, 1959). The tendency of the slurry flow in the channelisation process is to form the ideal section (semicircular) to minimise the energy required for the flow (i.e., minimum bed slope) but it is limited by the maximum slope and height at which particles can remain stable on the banks (the angle of repose).

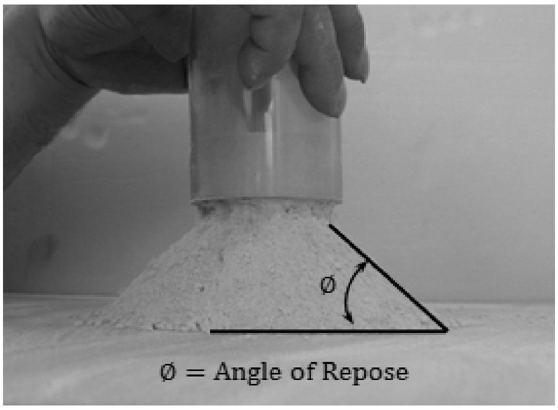
The US Bureau of Reclamation in 1951 developed a Stable Channel Profile for non-cohesive soils based on the theory of stable alluvial channel cross-section that can be found in many textbooks and research work (Simons and Senturk, 1992). The application of this model in its original form to self-formed channels of thickened tailings is presented in Pirouz and Williams (2007). A modified version of this model is introduced here, and is defined by the following set of equations. The only modification is the addition of the flat section with a width of T/2 and a depth of d_{max} for the central part of the section.

$$T = \pi \frac{d_{max}}{tan\,\emptyset}, A = \frac{d_{max}^2}{tan\,\emptyset}\left(2 + \frac{\pi}{2}\right) \tag{4}$$

$$R_H = \frac{\left(2+\frac{\pi}{2}\right) d_{max} \cos\emptyset}{2\left(E(\sin\emptyset) + \frac{\pi}{4}\cos\emptyset\right)}, E(\sin\emptyset) = \frac{\pi}{2}\left(1 - \frac{1}{4}\sin^2\emptyset\right) \tag{5}$$

Where $\emptyset$ is the angle of repose of the material, d_{max} is the maximum depth at the centreline of the channel, T is a parameter which defines the top width of the flow as shown in Figure 8, A is the cross-sectional area and R_H is the hydraulic radius of the section.

The angle of repose for oven-dried material can be estimated by the hollow cylinder method (Zhichao, 2011). The use of a plastic cylinder with a height to diameter ratio of 2.5 is recommended. The test method is shown in Figure 7. The measured values of the angle of repose for some tailings samples are also presented in Figure 7 as a reference.



Sample No.	Percentage Passing 75µm	Particle Size (mm)		Angle of Repose (Degree)
		D50	D85	
1	58.0	0.061	0.185	43.5
2	57.1	0.053	0.218	40.5
3	49.0	0.080	0.290	40.0
4	51.9	0.073	0.221	44.5
5	75.0	0.025	0.116	42.8

Figure 7 Measurement of angle of repose for dry tailings

Figure 8 shows the stable channel section model fit for one of the self-formed channel section profiles measured during testwork at the Chuquicamata mine site (Pirouz et al. 2013).

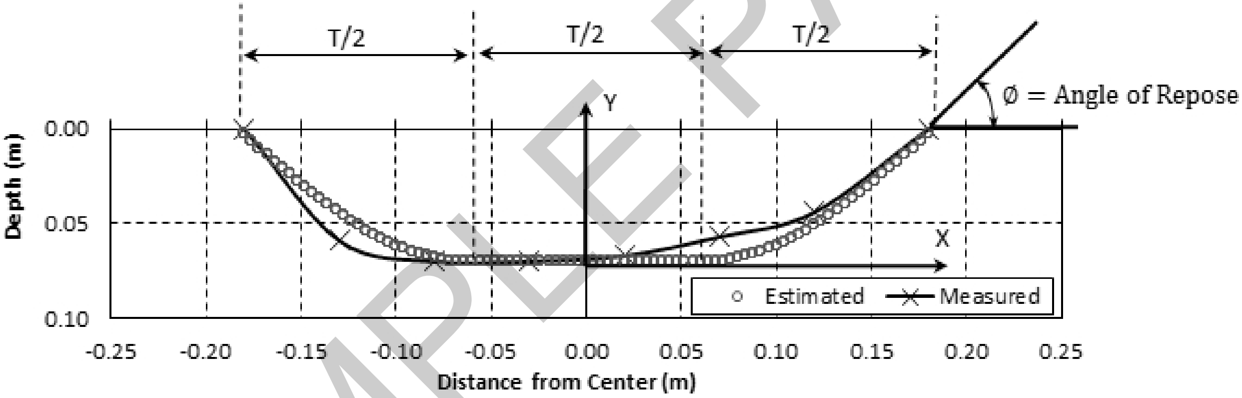


Figure 8 Stable channel section model fit to measured self-formed channel

3 Experimental data used in the development of the model

The hydrodynamics of non-Newtonian fluids is an extremely complex topic. The dominating effect of viscosity makes it impossible to use only theoretical methods to obtain desired results. The analysis of the flow condition and behaviour require accurate and reliable experimental data.

The experimental data used in this paper for the development of the beach slope prediction model are from previous research work carried out at Peak gold mine in Cobar, New South Wales, Australia (Pirouz et al., 2005, 2007), Sunrise Dam gold mine in Western Australia (Fitton et al. 2007) and Chuquicamata copper mine in Calama, Chile (Pirouz et al. 2013).

All of these research works were undertaken based on the equilibrium slope model and evaluating the tailings beach slope using an adjustable slope flume.

The details of the testing methodology and data acquisition are explained in Pirouz et al. (2013).

4 Structure of beach slope prediction model

Any methodology for the estimation of the equilibrium slope in an open channel for any solids concentration should provide proper answers to the following questions:

- What is the dominant solids transport mechanism in the flowing slurry?
- How much velocity (turbulence) is needed to keep the solids particles within the flow and to carry them using that particular transport mechanism?
- How much bed slope (energy gradient) is needed to create the required level of velocity (turbulence) to overcome all friction losses in the channel in order to maintain the flow?

The first question has been addressed in Section 2.3. In the following sections the analysis of the data collected from the tilting flume testing is used to answer two other questions.

4.1 How much turbulence is needed to overcome the zone settling?

The issue of minimum transport velocity in pipe flow of non-Newtonian fluids has been the subject of many research projects. Correlation equations exist for the so-called minimum transport velocity or minimum deposition velocity in pipe flow (Wasp and Slatter, 2004). All of the available equations are empirical and are based on experimental data gathered from pipe-loop testwork on material other than thickened tailings slurries. This makes them not applicable for the case of non-segregating thickened tailings. An empirical correlation equation for the minimum deposition velocity in channel flow of non-segregating slurries based on the limited data from the testwork at Sunrise Dam mine is presented by Fitton et al. (2007).

A different approach has been adopted in the current study by correlating the two dimensionless numbers, the Reynolds Re and Hedstrom He numbers, at the equilibrium slope condition. The Reynolds number is defined as the ratio of inertial forces to viscous forces. The Hedstrom number is the product of Reynolds number and Plasticity Index P_L and considers the effect of yield stress in non-Newtonian fluids flow (Hedstrom, 1952). These dimensionless numbers were originally developed for pipe flow and are expressed as:

$$Re = \frac{D\rho_m V}{\mu}, He = \frac{D^2 \rho_m \tau_y}{\mu^2}, P_L = \frac{\tau_y D}{\mu V} \tag{6}$$

Where *D* is the pipe diameter, *V* is the flow velocity and μ is the apparent viscosity of the fluid. The apparent viscosity is defined as:

$$\mu = \frac{\tau}{\dot{\gamma}} \tag{7}$$

Where $\dot{\gamma}$ is the shear rate and can be replaced by bulk shear rate $8V/D$. By substituting bulk shear rate in Equation (7), using Equation (1) to define the shear stress and replacing *D* with $4R_H$, the following forms of the Reynolds and Hedstrom numbers can be defined for the flow of a Herschel-Bulkley fluid in open channel:

$$Re_2 = \frac{8\rho_m V^2}{\tau_y + K\left(\frac{2V}{R_H}\right)^n}, He_2 = \frac{64\rho_m \tau_y V^2}{\left(\tau_y + K\left(\frac{2V}{R_H}\right)^n\right)^2} \tag{8}$$

The above definition for Reynolds number is given by Haldenwang et al. (2002). The following relationship between this new Hedstrom number He_2 and the Reynolds number Re_2 can be defined:

$$\mathrm{He}_2 = \frac{8\tau_y}{\tau_y + K\left(\frac{2V}{R_H}\right)^n} \times \mathrm{Re}_2 \tag{9}$$

A similar relationship between the Reynolds and Hedstrom number in their original form also exists since the Hedstrom number is the product of Plasticity number and Reynolds number (Abulnaga, 2002).

Figure 9 shows the plot of Re_2 versus He_2 for all of the tilting flume test data. It is seen that a strong correlation between these two dimensionless numbers exist for the flow of thickened tailings slurry in the equilibrium condition. It is also worth noting that this correlation only applies for non-segregating slurries. When the segregating data for Sunrise Dam testwork (Fitton et al., 2007) is added to the graph the points are scattered.

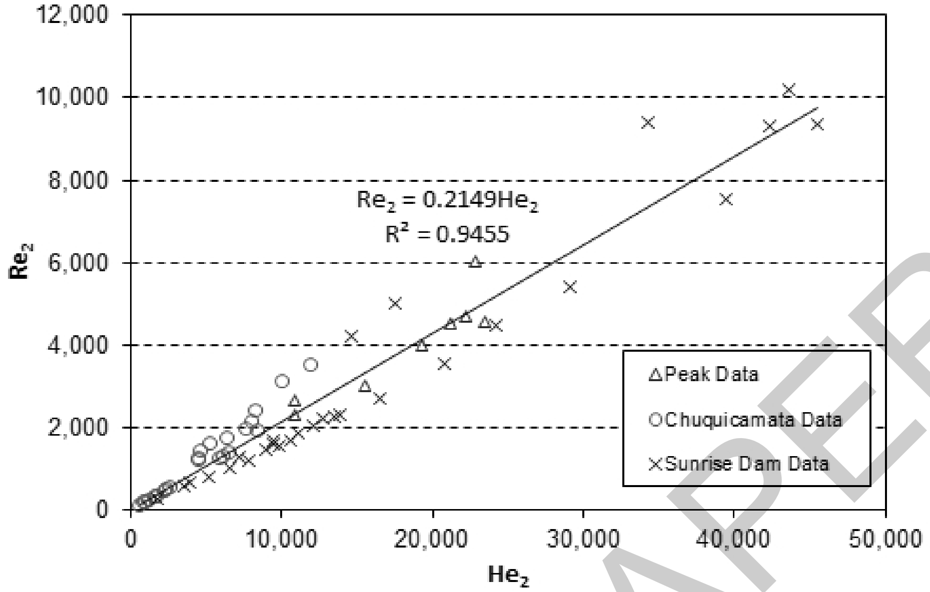


Figure 9 Variation of Reynolds number with Hedstrom number at equilibrium slope condition

One of the other very interesting findings is that this correlation exists in the equilibrium slope channel for all flow conditions regardless of the existence or non-existence of the plug.

A similar approach has been adopted in the past by researchers to define the transition from laminar to turbulent flow in pipes using the original form of Reynolds and Hedstrom numbers (Hanks, 1963; Slatter and Wasp, 2000).

The correlation between average flow velocity and Reynolds number Re_2 at equilibrium slope is presented in Figure 10. As seen in this figure, data are scattered for Reynolds numbers between 700 and 5,000 and the correlation between V_c and Re_2 is weak, which makes it difficult to accurately define the critical velocity V_c in this region using this approach.

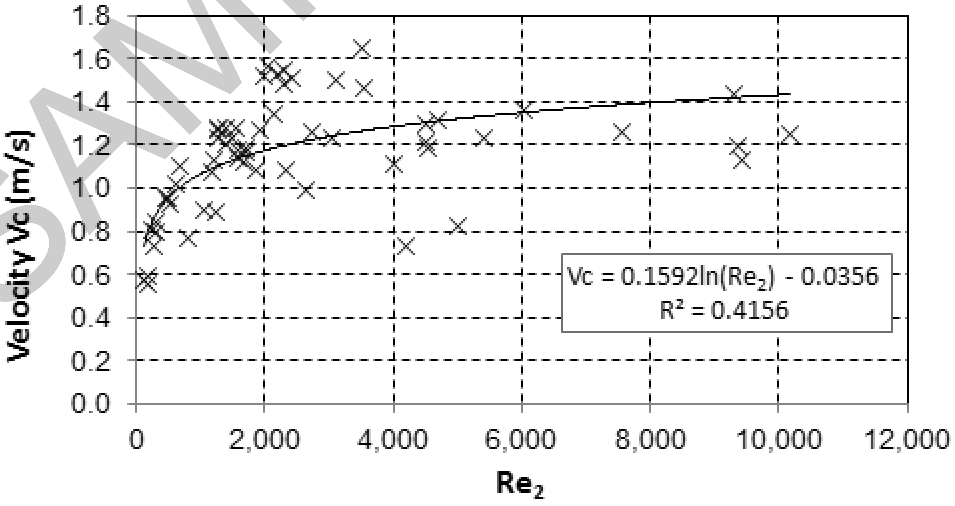


Figure 10 Variation of average channel velocity with Reynolds number

The strong correlation that exists between Re_2 and He_2 for channel flow at the equilibrium slope condition is a link between the rheological parameters of slurry (which have a direct impact on zone settling) and the flow characteristics (i.e., velocity). Hence, it is proposed by the authors that the correlation in Figure 9 be used for calculating the level of turbulence required in the channel. This answers the second question for the beach slope prediction model.

4.2 Head loss models and the problem with their application in non-Newtonian fluids

There are several mathematical models that have been developed over the years for calculation of energy loss (head loss) in both pipe and open channel flows. One the most commonly used equations, originally developed from the fundamental equations for pipe flow, is the well-known Darcy-Weisbach formula:

$$S_f = \frac{f_D V^2}{8 g R_H} \tag{10}$$

Where S_f is the channel slope, f_D is the Darcy friction factor, R_H is the hydraulic radius of the channel and $V$is the average velocity of flow in the channel.

The complication in using Equation (10) for non-Newtonian fluids in both pipe and open channel is being able to appropriately define the Darcy friction factor. Some of the available models for friction loss calculation in a non-Newtonian fluid are reviewed in Pirouz and Williams (2007). This review revealed that currently very limited theoretical models and published experimental data are available on this topic.

Figure 11 shows the plot of actual measured $f_D$in the flume tests versus measured Re_2. The estimated curves using some of the most comon friction factor models are also plotted for comparision. The short- coming of the existing models to predict f_D for thickened slurry flow is obvious, especially for the range of the Reynolds number between 500 and 2,500.

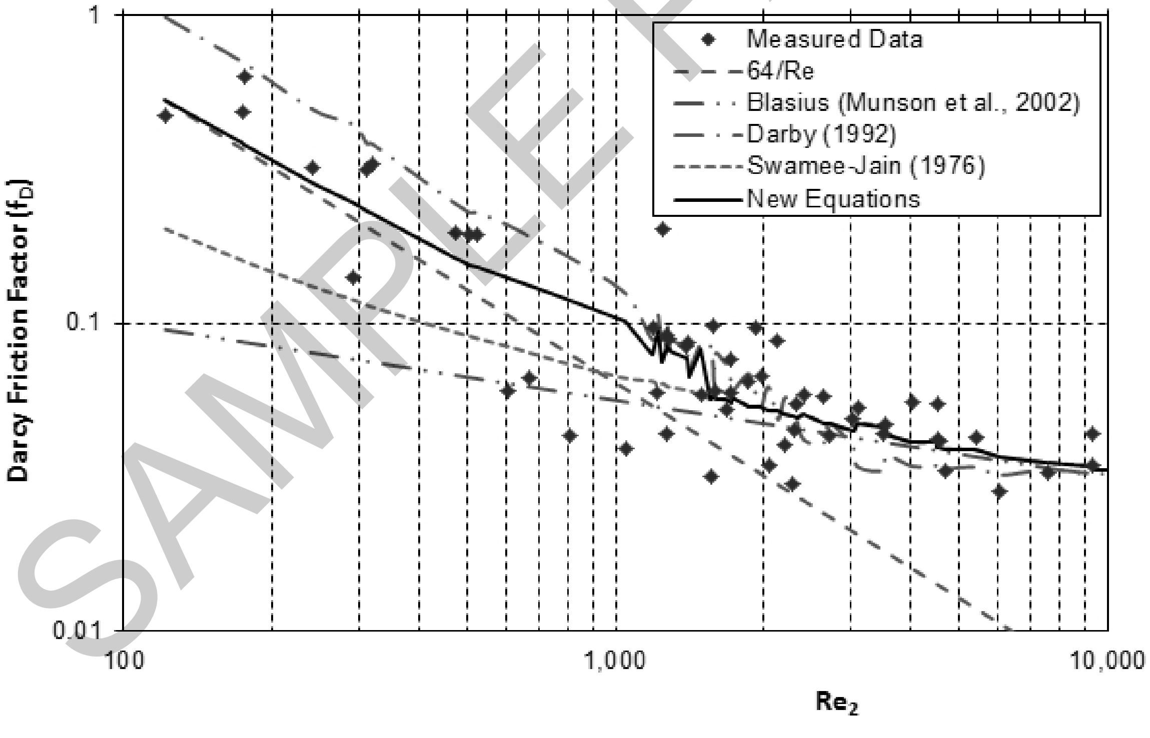


Figure 11 Variation of Darcy friction factor with Reynolds number Re_2

For the purpose of this study the following set of equations, which are the combination of Darby's friction factor (Darby et al., 1992) modified as per Equation 11 below for a low Reynolds number, and the Swamee-Jain equation (Swamee and Jain, 1976) for a high Reynolds number are proposed to be used as the best available estimate:

$$\text{Modified Darby Mode } f_D = \frac{64}{Re_2}\left(1+\frac{He_2}{6Re_2}\times\frac{Re_2-100}{2000}\right) \text{ if } Re_2 \le 1500 \quad (11)$$

$$\text{Swamee-Jain Model } f_D = \frac{0.25}{\left[log\left(\frac{k_S}{14.8R_H}+\frac{5.74}{{Re_2}^{0.9}}\right)\right]^2} \text{ if } Re_2 > 1500 \quad (12)$$

Where k_s is the particle roughness and may be taken as equal to D_{90} of the tailings.

The curve using Equations (11) and (12) is also plotted in Figure 11 (labelled as New Equation).

5 Beach slope prediction model

The following step-by-step procedure is proposed for the estimation of thickened tailings beach slope. The procedure is only applicable to non-segregating slurries.

5.1 Required Input data

Slurry discharge flow rate Q, density of slurry mix $\rho_{m,}$ rheological parameters (Herschel-Bulkley model parameters), angle of repose for oven dried samples ∅ (values presented in Figure 7 can be used as a reference), D_{90} for the tailings particles, segregation test results to confirm that the slurry at $\rho_m$is non-segregating.

5.2 Prediction procedure

1. Assume a value for maximum depth for the channel d_{max} (start with $d_{max} = 0.01m$).
2. Having d_{max} and the angle of repose for the dried solids ∅, calculate the channel cross-section area "*A*" and hydraulic radius "R_H" using Equations 4 and 5.
3. Having slurry flow rate and the channel cross-section area, calculate the average flow velocity ($V = Q/A$).
4. Using the average velocity "*V*" calculated in Step 3 together with the rheological parameters for the slurry and the hydraulic radius calculated in Step 2, calculate the Re_2 and He_2 from Equation 8.
5. Using He_2 from Step 4 calculate a new Re_2 from the following correlation equation:

$$Re_2 = 0.215\, He_2 \quad (13)$$

6. Compare the two Re_2 values calculated in steps 4 and 5. If these two values are equal go to Step 7. If not go back to Step 1. Assume a different value for d_{max} and repeat Steps 1 to 5 until these two Re_2 values from Steps 4 and 5 are the same. Then go to Step 7.
7. Having Re_2 , He_2 , *V*, R_H and also $k_s = D_{90}$ calculate the Darcy friction factor from Equations 11 or 12 (whichever is relevant, depending on the Re_2 value).
8. Use the calculated Darcy friction factor from Step 7, together with *V* and R_H to predict the beach slope from Equation 10.

   Note 1: The above procedure has been successfully applied to all of the flume test data, however it is found that for some slurry rheology (for unknown reasons) the above procedure does not find a stable solution and the calculated channel velocity "*V*" after the completion of Step 6 reaches very high values. A solution to this problem is as follows.

   After the completion of Step 6 if the calculated flow velocity is higher than 1.4 m/s or smaller than 0.8 m/s, go back to Step 1 and start the procedure again. This time skip Steps 5 and 6. Use the Re_2 from Step 4 to calculate 'Vc' from Equation 14 given below (which is from Figure 10), check 'Vc' with the average flow velocity calculated in Step 3. If $V = Vc$ go to Step 7. If not go back to Step 1.

Repeat Steps 1 to 4 and the procedure described in this note (skip Steps 5 and 6 each time) until $V = Vc$ then go to Step 7.

$$Vc = 0.1592 \ln(Re_2) - 0.0356 \tag{14}$$

5.3 Validation of prediction method

The procedure described in the previous section has been successfully applied to data from experimental flume testing. A comparison between predicted slope and the measured slope in the flume is given in Figure 12.

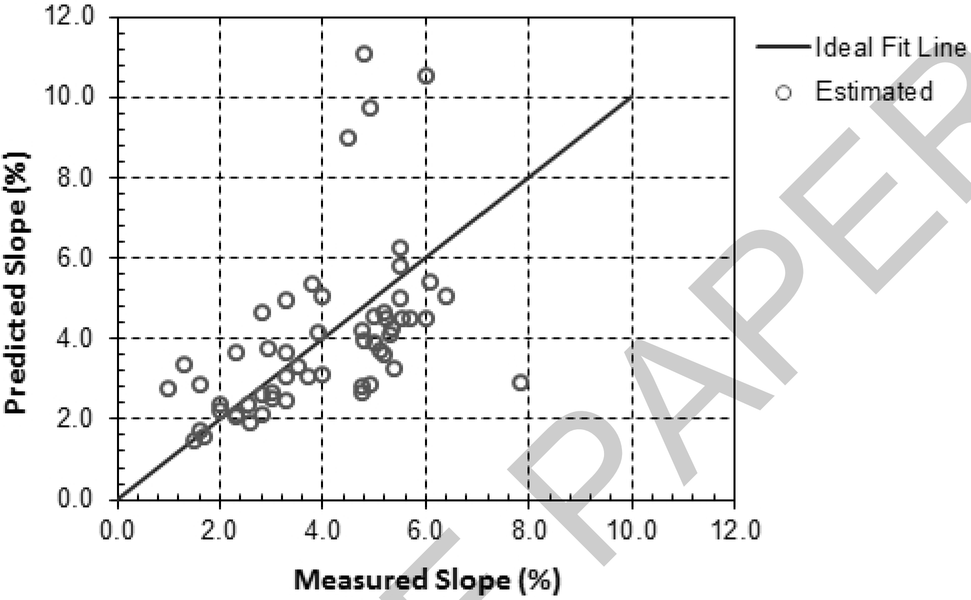


Figure 12 Estimated equilibrium slope versus measured slope for flume test data

As seen in Figure 12 there are a few points in the plot (four points above the Ideal fit line and one point below) that do not match well with the predicted values. It is assumed that this is due to inaccuracy in the measured data. Even when discarding these values, there is nevertheless considerable scatter, as much as between half to two times the predicted value. This is far from ideal, but it must be remembered that the area being looked at is the transition between laminar and turbulent flow. All previous experimenters, from Moody onwards, have encountered such uncertainty in this zone. It is intrinsically chaotic and unstable, and the introduction of a settling suspension rather than a purely homogeneous fluid adds to the problem. Further research can only improve the situation but, for now, the authors suggest that we just have to live with it and develop a design philosophy that is suitably robust despite the range of slope values that the model suggests might occur in practice.

Repeating this point, as presented in Figure 11, the existing models for estimation of the Darcy friction factor f_D are not able to estimate this key parameter accurately for the entire range of Reynolds number and flow condition. This is an area that needs further research and improvement.

An independent evaluation of the prediction method has also been carried out by using the data published by Seddon and Fitton (2011) for 5 different mines. The results are presented in Figure 13.

As can be seen, the estimated beach slopes for these mines are within the range of scatter from Figure 12.

Clearly, at this stage a reasonable contingency needs to be built into the design, and the consequences of the variation of beach slope within this range need to be evaluated and addressed properly at the study phase of the project.

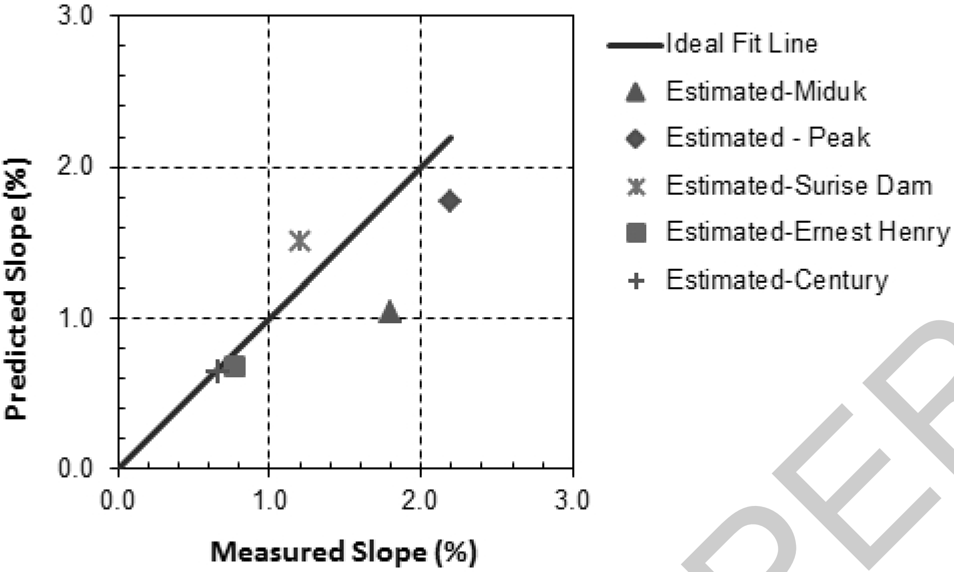


Figure 13 Estimated equilibrium slope versus measured slope for flume tests data

6 Conclusions

The observations from tilt flume testing at three different mine sites are used to develop a descriptive model for channelisation of slurry flow over the stack formed by thickened tailings discharge. Based on this model, a certain level of turbulence is required in the self-formed channels to counterbalance the zone-settling velocity of the particles and to maintain the required steady state, total transport condition.

For thicker slurries, the combined effect of yield stress and the bed slope required to overcome the zone settling may result in the formation of a central plug which is carried by a sheared zone.

Some of the recorded Reynolds numbers for the tilt flume testing are as low as 120, but even in this range of Reynolds numbers some level of turbulence must exist in the channel (or within the sheared zone).

The fact that for a viscous fluid the transition between laminar and turbulent regimes occurs at such a low Reynolds number has been observed by other researchers.

> *The more viscous the fluid became, the smoother the transition region became, and the lower the Reynolds numbers than the classical transition region around Re = 2,000. (Haldenwang and Slatter, 2006)*

An empirical relation between a modified Reynolds number (Re) and modified Hedstrom number (He) has been observed for tailings flow at the equilibrium slope. This unique relation between the two dimensionless numbers defines the condition of flow in the self-formed channel and can be used in the prediction of beach slope.

The experimental data from the tilting flume tests are used to develop a model for prediction of thickened tailings beach slope. The model was successfully validated with a set of independent data.

It was found that very little data are available for friction factor in open channel flow of viscous slurries. To the knowledge of the authors, the experimental data collected during the current research work (Pirouz and Williams, 2007; Pirouz et al. 2013; Fitton et al., 2007) are some of the very few published data sets on the flow of thickened tailings slurries in open channels.

The final, unavoidable conclusion is that this is a topic that remains in need of further research.

References

Abulnaga, B.E. (2002) Slurry systems handbook, Mazdak International Inc., New York, McGraw-Hill, pp. 6.42–6.43.

Chow, V.T. (1959) Open channel hydraulics, McGrawhill, New York. pp. 160–162.

Darby, R., Mun, R. and Boger, V. (1992) Prediction friction loss in slurry pipes, Chemical Engineering, September 1992.

Fitton, T.G., Williams, M.P.A., Seddon, K.D., Bhattacharya, S.N. and Chryss, A.G. (2007) Simulation of thickened tailings stacks, in Proceedings Tenth International Seminar on Paste and Thickened Tailings (Paste07), A.B. Fourie and R.J. Jewell (eds), 13–15 March 2007, Perth, Australia, Australian Centre for Geomechanics, Perth, pp. 305–313.

Haldenwang, R., Slatter, P.T. and Chhabara, R.P. (2002) Laminar and transition flow in open channel for non-Newtonian fluids, Hydrotransport 15: 15th International Conference on the Hydraulic Transport of Solids in Pipes, Banff, Canada, pp. 755–768.

Haldenwang, R. and Slatter, P.T. (2006) Experimental procedure and database for non-Newtonian open channel flow, Journal of Hydraulic Research, Vol. 44(2), pp. 283–287.

Hanks, R.W. and Pratt, D.R. (1967) On the flow of Bingham plastic slurries in pipe and between parallel plates, Society of Petroleum Engineers Journal, December 1967, pp. 342–346.

Hanks, R.W. (1963) The laminar-turbulent transition for fluids with a yield stress, A.I.Ch.E. Journal, May 1963, pp. 306–309.

Hedstrom, B.O.A. (1952) Flow of plastics materials in pipes, Industrial and Engineering Chemistry, Vol. 44,(3), pp. 651–656.

Pirouz, B., Kavianpour, M.R. and Williams, P. (2005) Thickened tailings beach deposition, field observations and full-scale flume testing, in Proceedings Eight International Seminar on Paste and Thickened Tailings (Paste05), R.J. Jewell and S. Barrera (eds), 20–22 April 2005, Santiago, Chile, Australian Centre for Geomechanics, Perth, pp. 53–72.

Pirouz B., Kavianpour M.R. and P. Williams, (2008), Sheared and un-sheared segregation and settling behaviour of fine sand particles in hyperconcentrated homogenous sand-water mixture flows, J Hydraulic Research, Vol. 46(1), pp. 105–111.

Pirouz, B., Seddon, K.D., Pavissich, C., Williams, P. and Echevarria, J. (2013) Flow through tilt flume testing for beach slope evaluation at Chuquicamata Mine CODELCO, Chile, in Proceedings 16th International Seminar on Paste and Thickened Tailings (Paste 2013), R.J. Jewell, A.B. Fourie, J. Caldwell and J. Pimenta (eds), 17–19 June 2013, Belo Horizonte, Brazil, Australian Centre for Geomechanics, pp. 459–474.

Pirouz, B. and Williams, P. (2007) Prediction of non-segregating thickened tailings beach slope – a new method, in Proceedings Tenth International Seminar on Paste and Thickened Tailings (Paste07), 13–15 March 2007, Perth, Australia, Australian Centre for Geomechanics, Perth, pp. 315–327.

Seddon, K.D. and Fitton, T.G. (2011) Realistic beach slope prediction and design, in Proceedings 14th International Seminar on Paste and Thickened Tailings (Paste2011), R.J. Jewell and A.B. Fourie (eds), 5–7 April 2011, Perth, Australia, Australian Centre for Geomechanics, Perth, pp. 281–294.

Simons, D.B. and Senturk F. (1992) Sediment transport technology, ISBN 0918334667, Water Resources Publications, Colorado, USA.

Slatter, P.T. (1997) The effect of the yield stress on laminar/turbulent transition, in Proceedings 9th International Conference on Transport and Sedimentation of Solids Particles, 2–5 September 1997, Carkow, Poland.

Slatter, P.T. and Wasp, E.J. (2000) The laminar/turbulent transition in large pipes, in Proceedings 10th International Conference on Transport and Sedimentation of Solid Particles, 4–7 September 2007, Wroclaw, pp. 389–399.

Swamee, P. and Jain, A. (1976) Explicit equations for pipe-flow problems, Journal of the Hydraulics Division (ASCE), Vol. 102(5), pp. 657–664.

Wasp, E.J. and Slatter, P.T (2004) Deposition velocities for small particles in large pipes, in Proceedings 12th International Conference on Transport and Sedimentation of Solid Particles, 20–24 September 2004, Prague, Czech Republic.

Williams, M.P.A. (2001) Tailings beach slope forecasting a review, in Proceedings High Density and Thickened Tailings Conference, Pilanesberg, South Africa.

Williams, M.P.A. (2009) Short course: An overview of thickened tailings technology, tailings disposal-beaching and depositional behaviour, 12th International Seminar on Paste and Thickened Tailings, 21–24 April 2009, Viña Del Mar, Chile.

Williams, M.P.A. and Meynink, W.J.C. (1986) Tailings beach slopes, Workshop on Mine Tailings Disposal, The University of Queensland, August 1986, pp. 30–61.

Zhichao, L. (2011) Measuring the angle of repose of granular system using hollow cylinders, MSc thesis, University of Pittsburgh, USA.